Low-Cost Adaptive LS-DEKF for SOC Estimation and RDT Prediction with SFP Model

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Abstract—Simplified first principle (SFP) lithium-ion battery model has high accuracy in state-of-charge (SOC) estimation and remaining-dischargeable-time (RDT) prediction. However, SFP model also has high computation and puts challenges for real-time applications. To this end, a Lebesgue sampling (LS) based method was proposed to reduce the computation of the SFP model without sacrificing the accuracy. The LS-based methods are event-based, in which algorithms are executed only when necessary. The main challenge of LS-based method is the low convergence speed of SOC estimation and RDT prediction, especially when the batteries are under dynamic loading profiles. This paper therefore proposes a low-cost solution that integrates adaptation of Lebesgue length and dichotomy with LS-based method, which is implemented with extended Kalman filter. The novelty of the proposed adaptive LS-based dichotomy with EKF (ALS-DEKF) is two-fold: (i) it fuses the dichotomy and EKF in LS framework to improve the accuracy and convergence speed; (ii) it optimizes the Lebesgue length according to the battery load to accommodate the dynamic loading profiles. The proposed ALS-DEKF is then verified with a series of experiments under dynamic operating conditions. Experiment results and comparison studies show that ALS-DEKF can achieve fast convergence speed and high accuracy under dynamic loading profiles for SOC estimation and RDT prediction.

Index Terms—Simplified first principle, Lebesgue sampling, dichotomy, extended Kalman filter, state-of-charge, remaining-dischargeable-time.

I. INTRODUCTION

Lithium-ion batteries (LIBs) are critical energy storage devices, and have been widely used from electric vehicles to ships and satellites, etc., for the advantages of high power and energy density, low self-discharge rate, long lifetime span, and no memory effect [1]–[3]. However, accurate and reliable state-of-charge (SOC) estimation and remaining-dischargeable-time (RDT) prediction are challenging, especially when LIBs work under dynamic loading profiles. SOC is one of the most important states to be monitored by battery management system (BMS) to optimize the performance and extend the life span of LIBs [4]. RDT prediction mainly provides information of the time that the battery can be in service in the future, which helps enhancing efficiency and effectiveness of decision-making, and preventing abnormal scenarios such as thermal runaway, short circuit fault and overcharge/discharge, and many others.

Many SOC estimation and RDT prediction methods are available. They can be classified as model-free and model-based methods. Model-free methods are based on heuristics, such as Coulomb counting approach [5], and data-driven approaches (artificial neural networks [6], support vector regression [7] or Gaussian process regression [8]). Model-based methods are integrated with the application of an observer (extended Kalman filter (EKF) derivatives [9], [10], particle filter (PF) [3], or sliding model [11]) to a battery model like electrochemical models (EMs) [12], [13], equivalent circuit models (ECMs) [14], [15], or curve-fitting models [16].

In real applications, model-based methods are mainly based on ECMs and EMs. Currently, most BMSs adopt ECMs for its simplicity. However, ECMs belong to empirical models and they cannot describe the electrochemical reaction processes inside batteries. In real applications, it is difficult to obtain reliable ECM parameters due to complex electrochemical reactions, nonlinear LIB dynamics, and varying operating conditions. As a result, the accuracy of SOC estimation and RDT prediction of ECM-based methods often cannot be guaranteed in the entire lifespan of LIBs. On the contrary, EMs, including pseudo two-dimensional (P2D) model [17], single particle (SP) model [9], and simplified first principle (SFP) model [2], [12], have abilities to depict the processes of mass conservation, energy balance, and electrochemical reaction kinetics. Thus, EMs have promising performance in describing the battery internal dynamics, which lead to better performance of EM-based methods. However, a major limitation of EM-based methods is high computational demand. Compared to ECMs, EMs have more state variables, parameters, and partial differential equations, which require more computational resources, especially in traditional Riemann sampling (RS)-based framework. The high computation demands put challenges on the implementation of EMs-based methods on embedded systems or microcontrollers for industrial applications.

To address these issues, an approach that integrates Lebesgue-sampling (LS) with EKF (LS-EKF) for states estimation and prediction was developed [18]. In this design, the LS-EKF algorithm is event-triggered, i.e., it is executed as needed [18]. Different from RS-based methods, LS-based methods divide the state axis into a number of predefined states (Lebesgue states). When a new measurement becomes available, a checker first checks if the measurement triggers an event, i.e., whether it reaches a new Lebesgue state. If yes, it indicates the state has significant changes and the algorithm is executed for new state estimation and prediction. If not, the algorithm is not executed and the estimated state is same as the one from the previous event. Besides, traditional battery diagnosis and prognosis algorithms [10], [19], [20] are implemented in RS framework, which usually need more computation resources, has slow state tracking speed, and
the prognosis uncertainties are usually large due to long-term noise accumulation. The proposed LS-based method has the advantages of high tracking speed, low computation, low uncertainty in state prediction, and good accuracy.

In previous LS-EKF, the Lebesgue length is selected as a constant. The constant-length LS-EKF methods, although work well for constant loading profile, is not optimal for dynamic loading profile. To accommodate dynamic loading profile, a feasible solution is to adjust the Lebesgue length adaptively to meet the requirement of accuracy of state estimation and prediction, and the demand for hardware.

Aligning with above motivations, this paper proposes an adaptive LS-based dichotomy with EKF (ALS-DEKF) approach. In this approach, the SOC changing speed (proportional to current loading profile) is used to adjust the Lebesgue length adaptively and obtains a new set of Lebesgue states on the whole SOC state axis. With this design, the new Lebesgue length increases (decreases) when the SOC changing speed increases (decreases) and is re-adjusted every time when the diagnosis is executed. This way, the algorithm can closely monitor the dynamics of SOC and RDT, even with high current profiles. In RDT prediction, the Lebesgue length at each Lebesgue state is adjusted according to the previous and current Lebesgue state information. The prognostic algorithm then predicts the RDT probability distribution function (PDF) on each updated Lebesgue state step by step. In ALS-DEKF, the dichotomy is used to calculate the a priori of SOC for EKF to improve the convergence speed of SOC estimation and RDT prediction. With the integration of LS-EKF and dichotomy, the proposed ALS-DEKF approach is able to estimate SOC and predicate RDT accurately with low computation and high convergence speed.

The contributions of the proposed method are summarized as follows: 1) develop an adaptive Lebesgue length method in the LS framework to accommodate the dynamic loading profiles and improve computation efficiency; 2) Integrate a dichotomy approach with EKF in LS for SOC estimation and RDT prediction based on SFP model to improve accuracy and convergence speed; and 3) Conduct a series of experiments under dynamic loading profiles for verification and validation.

This paper is organized as follows: Section II provides the SFP model in Lebesgue sampling framework. Section III provides SFP model integration with ALS-DEKF for SOC estimation and RDT prediction. Section IV presents a case study to demonstrate the advantages of ALS-DEKF. Finally, conclusions and future research are discussed in Section V.

II. Lebesgue Sampling Model

A. Lebesgue sampling

SOC in this research is defined as the ratio between the remaining usable charge and the total charge. SOC can be calculated by the Coulomb counting method as [21]:

$$soc_{t_k} = soc_{t_{k-1}} - \frac{\bar{I}_{\Delta t_k} \Delta t_k}{Q_{all}}$$  \hspace{1cm} (1)

where $soc_{t_k}$ is the SOC at $t_k$, $\bar{I}_{\Delta t_k}$ is the mean current in time interval $\Delta t_k$, $Q_{all}$ is the total capacity and can be measured by discharging the fully charged battery to the cutoff voltage.

Fig. 1 illustrates LS with a SOC degradation curve on which a number of Lebesgue states $[soc^m_{k-1}, soc^m_{k+1}, \cdots, soc^m_{EOD}]$, as illustrated by horizontal lines, are evenly defined on the SOC axis, where $soc^m_{EOD}$ is the Lebesgue state defined on the end-of-charge (EOD) threshold, and $D$ is the predefined Lebesgue length. The LS-based method is implemented only when the SOC measurement $soc^m$ (obtained from Coulomb counting method, and the superscript $m$ represents the measured value) changes from one Lebesgue state to another one, i.e., an event happens. This way, the frequency of SOC estimation depends on $soc^m$, which is determined by the loading profile of the battery. This design indicates that the algorithm executes more (less) often when the load is heavy (light). With this design, LS-based method is executed only when necessary and can greatly reduce the computation, while maintaining the accuracy of SOC estimation.
B. LS-based simplified first principle model

This section introduces the SFP model in LS framework that has high fidelity in describing battery behaviors from the micro-scale point of view. The parameters of SFP model are acquired from [2] and are optimized by particle swarm optimization (PSO) [23] to fit the SFP model with experiment data. Note that the experiments are conducted at room temperature, and the thermal coupling in the model is ignored.

1) Open circuit voltage: The open-circuit voltage (OCV) is described as [2]:

\[
E_{\text{ocv,} \, t_k} = U_p(y_{\text{surf,} \, t_k}) - U_n(x_{\text{surf,} \, t_k})
\]

where subscript \( t_k \) indicates the event time stamp. \( E_{\text{ocv}} \) is the open-circuit voltage; \( U_p \) and \( U_n \) are open circuit potential of positive and negative electrodes, respectively; \( y_{\text{surf}} \) and \( x_{\text{surf}} \), with initial values of \( y_0 \) and \( x_0 \), respectively, are solid-phase surface concentration of particles of positive and negative electrodes, respectively. After the LIB rests for a certain period of time, \( y_{\text{surf}} \) and \( x_{\text{surf}} \) are equal to the average solid-phase concentration \( y_{\text{avg}} \) and \( x_{\text{avg}} \) of positive and negative electrodes, respectively, which are described as [2]:

\[
y_{\text{avg,} \, t_k} = y_{\text{avg,} \, t_k-1} + \frac{I_{\text{avg,} \, \Delta t_k}}{Q_p} \Delta t_k
\]

\[
x_{\text{avg,} \, t_k} = x_{\text{avg,} \, t_k-1} - \frac{I_{\text{avg,} \, \Delta t_k}}{Q_n} \Delta t_k
\]

where \( I_{\Delta t_k} \) is the current in \( \Delta t_k \), with the sign being considered positive for discharge and negative for charge, \( Q_p \) and \( Q_n \) are capacities of the effective active materials of positive and negative electrodes, respectively, and \( \Delta t_k = t_k - t_{k-1} \) is the time between two events. 

2) Solid-phase diffusion: When a LIB works with a load current \( I \), the solid-phase diffusion process happens and results in concentration difference in particles of positive and negative electrodes, which are denoted as \( \Delta y \) and \( \Delta x \), respectively, and can be described as [2]:

\[
\Delta y_{\text{avg,} \, t_k} = \Delta y_{\text{avg,} \, t_k-1} + \frac{2 \tau_{\text{sp}}}{Q_p} \Delta t_k
\]

\[
\Delta x_{\text{avg,} \, t_k} = \Delta x_{\text{avg,} \, t_k-1} + \frac{2 \tau_{\text{sn}}}{Q_n} \Delta t_k
\]

in which \( \Delta y_{\text{avg,} \, t_k} \) and \( \Delta x_{\text{avg,} \, t_k} \) satisfy [2]:

\[
\Delta y_{\text{avg,} \, t_k} = \Delta y_{\text{avg,} \, t_k-1} + \frac{2 \tau_{\text{sp}}}{Q_p} \Delta t_k
\]

\[
\Delta x_{\text{avg,} \, t_k} = \Delta x_{\text{avg,} \, t_k-1} + \frac{2 \tau_{\text{sn}}}{Q_n} \Delta t_k
\]

where \( \tau_{\text{sp}} \) and \( \tau_{\text{sn}} \) are the solid-phase diffusion time constants of the positive and negative electrode, respectively.

3) Liquid-phase diffusion: The electrolyte overpotential \( \eta_e \) is directly determined by the Li-ions concentration at the current collectors of both electrodes, which is given as [2]:

\[
\eta_{e, \, t_k} = \frac{2RT}{F} \ln \left[ \frac{c_0 + \Delta c_{t_k}}{c_0 - \Delta c_{t_k}} \right]
\]

where \( R \) is the ideal gas constant; \( T \) is the temperature; \( F \) is the Faraday constant; \( t_+ \) is the transport number of the Li-ions; \( c_0 \) is the initial liquid-phase Li-ions concentration, and \( \Delta c \) is the electrolyte concentration variations from \( c_0 \) caused by liquid-phase diffusion, where \( \Delta c \) satisfies [2]:

\[
\Delta c_{t_k} = \Delta c_{t_{k-1}} + \frac{1}{\tau_{e}} \left[ P_{\text{con}} I_{\Delta t_k} - \Delta c_{t_{k-1}} \right] \Delta t_k
\]

where \( P_{\text{con}} \) and \( \tau_e \) are the proportion coefficient and liquid-phase diffusion time constant, respectively.

4) Reaction polarization: According to Butler-Volmer kinetics, the reaction polarization overpotential \( \eta_r \) is [2]:

\[
\eta_{r, \, t_k} = \frac{2RT}{F} \ln \left[ \sqrt{m_{n, \, t_k} + 1} + m_{n, \, t_k} \right]
\]

\[
+ \ln \left[ \sqrt{m_{p, \, t_k} + 1} + m_{p, \, t_k} \right]
\]

(8)

where \( P_{\text{act}} \) and \( P_{\text{con}} \) are the reaction polarization coefficients of positive and negative electrodes, respectively, which reflect the electrochemical reaction rate inside the battery.

5) Ohmic polarization: The ohmic polarization overpotential \( \eta_{\text{ohm}} \) is given as:

\[
\eta_{\text{ohm,} \, t_k} = R_{\text{ohm}} I_{\Delta t_k}
\]

(9)

where \( R_{\text{ohm}} \) is a lumped parameter related to the ohmic polarization, which can be directly measured.

6) Brief summary: From the above-mentioned internal electrochemical processes, the battery terminal voltage \( U_{T, \, t_k} \) can be calculated as:

\[
U_{T, \, t_k} = E_{\text{ocv,} \, t_k} - \eta_{e, \, t_k} - \eta_{r, \, t_k} - \eta_{\text{ohm,} \, t_k}
\]

(10)

The model contains 11 parameters, i.e., \( x_0, y_0, Q_p, Q_n, \tau_{\text{sp}}, \tau_{\text{sn}}, P_{\text{con}}, P_{\text{act}}, P_{\text{ohm}} \), and \( R_{\text{ohm}} \), which need to be identified for batteries under study.

C. Lebesgue time model

RDT can be defined as the discharge end time when SOC drops below the SOC threshold. The LTM is needed to predict the RDT distributions on the current Lebesgue state to all other future Lebesgue states. For fully charged batteries, the average solid-phase concentration of positive and negative electrodes at \( t_{k-1} \) are:

\[
y_{\text{avg,} \, t_{k-1}} = y_0 + \frac{(Q_{\text{all}})}{Q_p} \left( 1 - \frac{\text{soc}_{t_{k-1}}}{1} \right)
\]

\[
x_{\text{avg,} \, t_{k-1}} = x_0 - \frac{(Q_{\text{all}})}{Q_n} \left( 1 - \frac{\text{soc}_{t_{k-1}}}{1} \right)
\]

(11)

From (11), we have:

\[
y_{\text{avg,} \, t_k} - y_{\text{avg,} \, t_{k-1}} = -(Q_{\text{all}}/Q_p) (\text{soc}_{t_k} - \text{soc}_{t_{k-1}})
\]

\[
x_{\text{avg,} \, t_k} - x_{\text{avg,} \, t_{k-1}} = (Q_{\text{all}}/Q_n) (\text{soc}_{t_k} - \text{soc}_{t_{k-1}})
\]

(12)

Combining Eqs. (3) and (12), we have:

\[
I_{\Delta t_k} \Delta t_k = -Q_{\text{all}} \left( \text{soc}_{t_k} - \text{soc}_{t_{k-1}} \right)
\]

(13)

By noting \( \Delta t_k = t_k - t_{k-1} \), from Eq. (13) we have,

\[
t_k = t_{k-1} + Q_{\text{all}} (\text{soc}_{t_{k-1}} - \text{soc}_{t_k})/I_{\Delta t_k}
\]

(14)

Fig. 2 shows the LTM development from SFP model. When \( \text{soc}_{t_k} \) triggers a new event at the Lebesgue state \( \text{soc}_{t_k} \) at \( t_k \) and LS-EKF is executed, the estimated SOC PDF has the mean of \( \text{soc}_{t_k} \) (solid magenta distribution). The SOC PDF is then converted into a time distribution with mean of \( t_k \) (solid green distribution) [24]. The difference between \( \text{soc}_{t_k} \) and the last triggered Lebesgue state \( \text{soc}_{t_k} \) is:

\[
\delta_{t_k} = \text{soc}_{t_k} - \text{soc}_{t_k}
\]

(15)
From Eq. (1), we have:

\[
\delta_{tk}^L = \delta_{soc_k}^L Q_{all}/\bar{I}_{\Delta t_k}
\]  \hspace{1cm} (16)

where \(\delta_{tk}^L\) is the difference between \(t_k\) and \(t_k^L\) (the mean value of the predicted time PDF defined on \(soc_k^L\)).

SFP model integration with ALS-DEKF for SOC estimation and RDT prediction.

For ALS-DEKF implementation, the SOC estimation model in LS is given as:

\[
soc_{tk} = soc_{tk-1} - (P_{q} \bar{I}_{\Delta t_k} \Delta t_k)/Q_{all} + \omega_{d,tk}
\]  \hspace{1cm} (20)

where \(P_{q}\) is a hyperparameter; \(\omega_{tk}\) is the Gaussian process noise with zero-mean value and covariance \(Q_{tk}\).

The observation model is given as:

\[
U_{T,t_k}^{nim} = h_{SFP}(\bar{I}_{\Delta t_k}, y_{avg,t_k}, x_{avg,t_k}) + v_{d,t_k}
\]  \hspace{1cm} (21)

where \(v_d\) is the noise of observation model with zero-mean value and covariance \(R_{tk}\); \(h_{SFP}(\cdot)\) is a lumped nonlinear function from SFP model that describes the dynamics of terminal voltage under the influence of loading current and internal variables such as \(y_{avg}\) and \(x_{avg}\).

A. DEKF for SOC estimation

When EKF is used for SOC estimation, the convergence speed is usually not fast enough for high-response-requirement systems. For dichotomy-based SOC estimation, the accuracy heavily depends on the state estimation model and observation model, whose fidelity is usually impacted by many factors, such as ambient temperature, working conditions, physical characteristics, and many others. Besides, the dichotomy algorithm will fall into the local optimum when the threshold is not set properly or the model used for SOC estimation is not accurate enough. With the consideration of the high fidelity of the SFP model (20) and (21) and the characteristics of these two methods, the EKF is fused with dichotomy (noted as DEKF) to improve the convergence speed and the accuracy of SOC estimation.

III. SFP MODEL INTEGRATION WITH ALS-DEKF

EKF are widely used for SOC estimation due to its simplicity and good performance in dealing with nonlinear models. However, traditional EKF-based methods often show low convergence speed. To address this issue, this research integrates the dichotomy with EKF in LS to improve the convergence speed, computing efficiency, and accuracy. Fig. 3 shows the SFP model integration with ALS-DEKF for SOC estimation and RDT prediction.

As shown in Fig. 3, the DEKF algorithm includes three steps: the a priori SOC prediction, the revision of the a priori SOC by dichotomy, and update of the revised a priori SOC. First, the a priori SOC state at the next Lebesgue state is calculated based on model (20), and the internal battery state such as \(y_{avg}\) and \(x_{avg}\) are obtained from (11) to calculate \(U_{T}^{nim}\)
based on model (21). Second, the a priori SOC is revised by dichotomy to ensure the revised a priori SOC state is close to the ground truth. Third, the revised SOC state is updated by using the terminal voltage error \( U_{\text{sim}}^k - U_T^k \).

It is worthy noting that the voltage measurement error is obtained by the difference between the simulated voltage from the SFP model \( U_{\text{sim}}^k \) given by (21) and the measured terminal voltage \( U_T^k \). The dichotomy takes advantage of the high fidelity of the SFP model to update the a priori SOC state by minimizing the error between \( U_{\text{sim}}^k \) and \( U_T^k \). The updated a priori SOC state obtained from dichotomy is closer to the ground truth, which theoretically benefits the SOC convergence speed and estimation accuracy. In addition, the initial value of SOC represented by \( \text{soc}_0 \) is usually unknown, and it can be set to a random number between 0 and 1. Here, subscript \( t_0 \) in Fig. 3 represents the time for initialization when the system is initialized.

When an Lebesgue event occurs, the DEKF is executed. In DEKF, the nonlinear function \( h_{\text{SFP}}(\cdot) \) is linearized to calculate the Jacobian matrix \( H_t_k|_{\text{soc}_k} \), which is given as [26]

\[
H_{t_k|_{\text{soc}_k}} = \frac{U_{\text{sim}}^{t_k} (\text{soc}_k + \delta \text{soc}) - U_{\text{sim}}^{t_k} (\text{soc}_k)}{\delta \text{soc}}
\]  

(22)

where \( \delta \text{soc} \) is usually less than 0.01 to guarantee the accuracy of Jacobian matrix; \( H_{t_k|_{\text{soc}_k}} \) is the derivative of the simulated terminal voltage \( U_{\text{sim}}^{t_k} \) with respect to SOC at \( t_k \).

The equations for the update step are expressed as:

\[
P_{t_k|t_k-1} = A_{t_k} P_{t_k-1|t_k-1} A_{t_k}^T + Q_{t_k-1}
\]

(23)

\[
K_{t_k} = P_{t_k|t_k-1} H_{t_k}^T (H_{t_k} P_{t_k|t_k-1} H_{t_k}^T + R_{t_k})^{-1}
\]

(24)

\[
\hat{x}_{t_k|t_k} = (I - K_{t_k} H_{t_k}) P_{t_k|t_k-1}
\]

(25)

\[
x_{t_k|t_k} = x_{t_k|t_k-1} + K_{t_k} \left[ y_{t_k} - h(x_{t_k|t_k-1}, u_k) \right]
\]

(26)

where \( A_{t_k} \) is the Jacobian of the SOC state estimation model; \( K_{t_k} \) is the Kalman gain; \( R_{t_k} \) is the covariance matrix of observation noise \( v_{t_k} \), which satisfies \( E(v_{t_k} v_{t_k}^T) = R_{t_k} \); \( P_{t_k|t_k} \) is the updated covariance estimate; \( x_{t_k|t_k} \) is the updated SOC estimation; \( I \) is the identity matrix.

Fig. 4 shows the DEKF-based SOC estimation procedure in LS framework, which is described as follows:

**Fig. 4.** The revision of the a priori SOC by dichotomy

**Step 1:** Set initial state \( \text{soc}_0 \) and \( P_{t_0}, Q_{t_0}, R_{t_0} \), and the error threshold for terminal voltage as \( U_{T_{\text{TH}}} \).

**Step 2:** Calculate the a priori \( P_{t_k|t_k-1} \) and the a priori \( \text{soc}_{t_k|t_k-1} \) according to (20);

**Step 3:** Calculate \( y_{\text{avg},t_k} \) and \( x_{\text{avg},t_k} \) according to (11) by using the a priori estimation \( \text{soc}_{t_k|t_k-1} \);

**Step 4:** Calculate error \( e_{t_k} \) between \( U_{\text{sim}}^{t_k} \) and \( U_T^k \);

**Step 5:** Determine whether \( e_{t_k} \) is less than \( U_{T_{\text{TH}}} \), if no, go to step 2 to revise \( \text{soc}_{t_k|t_k-1} \). Otherwise, the a priori SOC from the last previous step is used in the EKF for the a posteriori SOC based on the terminal voltage measurement \( U_T \);

**Step 6:** Calculate Jacobian \( H_{t_k}, K_{t_k} \) and \( P_{t_k|t_k+1|t_k} \) based on Eqs. (22), (23), (24) and (25);

**Step 7:** With the revised \( \text{soc}_{t_k|t_k-1} \), update Kalman gain \( K_{t_k} \) to get the a posteriori estimation \( \text{soc}_{t_k|t_k} \). Let \( k = k + 1 \).

Note that \( U_{T_{\text{TH}}} \) depends on actual application requirements. If \( U_{T_{\text{TH}}} \) is too small, the dichotomy algorithm takes more time or may fall into local optimization. On the contrary, a large \( U_{T_{\text{TH}}} \) may decrease the efficiency of dichotomy, and therefore, the setting of \( U_{T_{\text{TH}}} \) is a trade-off balance.

**B. Adjustment of Lebesgue state length**

With dynamic loading profiles, the SOC changing speed is high when LIBs work under heavy loads, which requires SOC and RDT update more frequently to enable safe system operation. Therefore, the Lebesgue length should be update adaptively and optimally according to the SOC value and its changing rate, i.e., the working condition. In ALS-DEKF, the Lebesgue length (denoted as \( D_{t_k} \)) is online adjusted in both SOC estimation and RDT prediction processes. Fig. 5 shows the Lebesgue length \( D_{t_k} \) adaptation scheme illustrated with a battery SOC degradation data [27].

**Fig. 5.** Lebesgue length adaptation scheme in SOC estimation process

Suppose the SOC estimation algorithm is working on the current Lebesgue state \( \text{soc}_{t_k}^L \) with the Lebesgue state length \( D_{t_k} \) and \( D_{t_k-1} \), and the time intervals \( \Delta T_{t_k} \) and \( \Delta T_{t_k-1} \), while \( D_{t_k+1} \) is the Lebesgue state length of being adjusted for the next SOC estimation step. The slope \( S_k \) (expressed by the ratio of the Lebesgue state lengths and the time intervals) represents the nonlinearity of SOC growth in the past two events, which is proportional to the input current of the battery. The SOC changing speed in \( \text{soc}_{t_k}^L \) and \( \text{soc}_{t_k}^L \) is compared with the current Lebesgue state to determine the Lebesgue length for SOC estimation in the next step. If the SOC changing speed becomes faster (slower), the Lebesgue length for the
next SOC estimation is decreased (increased). The Lebesgue length $D_{t_k+1}$ for the next step is expressed as:

$$D_{t_k+1} = \frac{D_{t_k}/\Delta T_k}{D_{t_k-1}/\Delta T_{k-1}} = D_{t_k} \cdot \frac{\Delta T_{k-1}}{\Delta T_k}$$  \hspace{1cm} (27)

With (27), the Lebesgue length is adjusted iteratively according to the history information of SOC changing speed, which is more reasonable for dynamic current profiles.

Note that the SOC state estimation result at the current time instant $t_k$ is a state PDF, which cannot be used as the initial time PDF for RDT prediction in LTM. Therefore, the SOC PDF at the current time instant $t_k$ needs to be converted to its corresponding time PDF, which is then used as the initial time PDF for RDT prediction. To address this problem, the current Lebesgue state is treated as a pseudo threshold, and the time PDF on the current Lebesgue state can be calculated from the SOC PDF at the last time instant $t_{k-1}$ according to the law of total probability [24].

C. ALS-DEKF based method

1) ALS-DEKF based SOC estimation: SOC estimation aims to monitor the usable charge of the battery. The SOC state is obtained by applying Bayesian estimation with the SOC estimation model (20) and observation model (21). The implementation of ALS-DEKF based SOC estimation is described as follows:

Step 1: Divide the SOC state range (from 1 to 0) into a number of Lebesgue states according to the real-time adjusted $D_{t_k}$, with which the SOC estimation model is discretized in LS form (20).

Step 2: If SOC measurement $soc^m$ reaches a newly defined Lebesgue state, the estimation algorithm is triggered. Otherwise, the estimation results keep unchanged.

Step 3: Compare the estimated SOC PDF against the SOC threshold to determine whether the algorithm is terminated.

2) ALS-DEKF based RDT prediction: Different from RS-based SOC prediction, the ALS-based RDT prediction is conducted along the SOC state axis to calculate the time PDF on all future Lebesgue states with the LTM (18). The implementation of ALS-based RDT prediction is as follows:

Step 1: If the diagnostic algorithm is triggered and the SOC state PDF is estimated, the SOC state PDF is converted to a time PDF according to the law of total probability [24]. This time PDF is used as the initial value for LTM.

Step 2: With (18), the time PDF is projected recursively with $D_{t_k}$ to all future Lebesgue states. RDT is obtained on the Lebesgue state defined on the EOD threshold.

D. A brief summary of SOC estimation and RDT prediction

In the process of SOC estimation and RDT prediction, the Lebesgue length is adjusted until the Lebesgue state reaches the EOD threshold $soc_{EOD}$. With adjustment of Lebesgue states length in one step forward prediction from the Lebesgue state $soc_k^L$ to $soc_{k+1}^L$, the adjusted Lebesgue length $D_{t_k+1}$ is used for the following RDT prediction process from $soc_{k+1}$ to $soc_{EOD}$. This way, the whole set of Lebesgue states length is adjusted from $soc_k^L$ to $soc_{EOD}$ and becomes the initial condition for the next SOC estimation and RDT prediction. By conducting the adjustment of Lebesgue states length, the RDT prediction algorithm is executed with different frequencies according to the SOC changing speed (current value), which meets practical application requirements better.

IV. EXPERIMENTAL RESULTS

This section verifies the SFP model and demonstrates the proposed ALS-DEKF method with battery samples of different types, capacities, and ages. The proposed ALS-DEKF method is compared against the LS-based method to illustrate its advantages. The battery specification of Sanyo 14500 LiCoO2/graphite LIB from the battery samples is used as an example for discussion and analysis, which is summarized in Table I. Experimental data are automatically generated by the Arbin BT2000 system, Fig. 6, at 1 Hz sampling rate under room temperature. The SFP model parameter ranges are obtained from the literature [2] and are given in Table II.

In general, the higher the sampling frequency, the better the accuracy of state estimation, which is more beneficial to performance. However, higher sampling frequency will also increase the computation. The sampling frequency is usually determined by the requirements of hardware and electrochemical reaction processes of the battery. In order to balance the accuracy and the computation cost, the sampling frequency is selected as 1 Hz, which meets most of the requirements in real applications. For real battery applications, the proposed method is affected by many factors, including temperature, state of health (SOH), Coulombic efficiency, humidity, etc. Due to space limitation, the effects of these factors are not considered in this paper, and will be our future work.

Fig. 6. Arbin BT2000 test system used for experiments

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>BATTERY SPECIFICATIONS OF LICO O2 14500 BATTERY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specifications</td>
<td>Value</td>
</tr>
<tr>
<td>Height</td>
<td>50.0 $\pm$ 0.2 mm</td>
</tr>
<tr>
<td>Diameter</td>
<td>14.0 $\pm$ 0.2 mm</td>
</tr>
<tr>
<td>Weight</td>
<td>Approx.: 50 g</td>
</tr>
<tr>
<td>Rated capacity</td>
<td>840 mA h</td>
</tr>
<tr>
<td>Cutoff voltage of charging</td>
<td>4.2 V</td>
</tr>
<tr>
<td>Positive material</td>
<td>LiCoO2</td>
</tr>
<tr>
<td>Negative material</td>
<td>Graphite</td>
</tr>
</tbody>
</table>
TABLE II
PARAMETER RANGE FOR LiCoO2 14500 battery

<table>
<thead>
<tr>
<th>Model parameter</th>
<th>Parameter range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_0$ (−)</td>
<td>0.7047, 0.8613</td>
</tr>
<tr>
<td>$y_0$ (−)</td>
<td>0.3995, 0.4883</td>
</tr>
<tr>
<td>$Q_n (C)$</td>
<td>[3.1726, 3.8776]×10³</td>
</tr>
<tr>
<td>$Q_p (C)$</td>
<td>[5.1386, 6.2805]×10³</td>
</tr>
<tr>
<td>$\tau_{sn} (s)$</td>
<td>158.46, 193.68</td>
</tr>
<tr>
<td>$\tau_{sp} (s)$</td>
<td>103.85, 126.93</td>
</tr>
<tr>
<td>$\tau_e (s)$</td>
<td>53.88, 65.86</td>
</tr>
<tr>
<td>$P_{con} (mol m^{-3} A^{-1})$</td>
<td>[864.23, 1056.28]</td>
</tr>
<tr>
<td>$P_{act} (m^{-1.5} mol^{0.5} s)$</td>
<td>[5.2623, 6.7854]×10⁵</td>
</tr>
<tr>
<td>$P_{actp} (m^{-1.5} mol^{0.5} s)$</td>
<td>[6.4682, 7.9056]×10⁵</td>
</tr>
<tr>
<td>$R_{ohm} (\Omega)$</td>
<td>0.0609, 0.0745</td>
</tr>
</tbody>
</table>

### A. Model validations

With the parameters in Table II and the experimental data from Arbin system, model validation is conducted by comparing the simulation and experimental data of battery voltage. Fig. 7 shows the measured and simulated data of fresh Sanyo 14500 Li-CoO2/graphite/840 mAh under dynamic stress test (DST) as an example, and the average absolute errors (AAEs) of simulated and measured terminal voltage is 15.6 mV.

### B. Comparison results of SOC estimation

Fig. 8 shows SOC estimation results by RS-EKF, LS-EKF, and ALS-DEKF (marked in green, black, magenta line, respectively) under dynamic current load. Since the initial SOC is assumed unknown, it is randomly selected as $soc_{t0} = 60\%$. Compared with the convergence time of 483 seconds and 534 seconds without dichotomy (RS-EKF and LS-EKF), the SOC convergence time of ALS-DEKF is only 2 seconds, which improves the speed by $\frac{483 - 2}{483} = 99.59\%$ and $\frac{534 - 2}{534} = 99.63\%$, respectively. In RS-EKF, the AAE and maximum absolute errors (MAE) of SOC estimation is 2.51% and 4.94%, respectively. In LS-EKF, the AAE and the MAE between the estimated SOC and the measured SOC is 2.53% and 5.62%, respectively. In ALS-DEKF, the AAE and MAE of SOC estimation is 1.43% and 3.98%, respectively. The results show that ALS-DEKF can significantly improve the performance in terms of SOC estimation accuracy and convergence speed under dynamic current load during the whole discharging range.

### C. Comparison results of RDT prediction

In RDT prediction, the time distribution is recursively calculated on each future Lebesgue state and RDT distribution is obtained on the Lebesgue state defined on the EOD threshold. Fig. 9 compares the RDT prediction results in terms of $\alpha−\lambda$ metrics [28]. In terms of convergence speed of RDT prediction, it shows that ALS-DEKF (magenta line) converges faster than RS-EKF (green line) and LS-EKF (black line). In terms of RDT prediction accuracy, ALS-DEKF tracks the true EOD time closer and more steady than RS-EKF and LS-EKF. In summary, ALS-DEKF shows better prediction accuracy and convergence speed than RS-EKF and LS-EKF in the whole discharging process. This demonstrates that ALS-DEKF can achieve higher accuracy and fast convergence speed.
LS-EKF in accuracy and convergence speed of SOC estimation and RDT prediction.

<table>
<thead>
<tr>
<th>PERFORMANCE COMPARISON OF DIFFERENT METHODS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>SOC(_{CT}) (s)</td>
</tr>
<tr>
<td>SOC(_{AAE})</td>
</tr>
<tr>
<td>SOC(_{MAE})</td>
</tr>
<tr>
<td>RDT(_ {AAE}) (s)</td>
</tr>
<tr>
<td>PT (s)</td>
</tr>
</tbody>
</table>

\(SOC\(_{CT}\)\): SOC estimation convergence time given by second; \(SOC\(_{AAE}\)\): AAE of SOC estimation; \(SOC\(_{MAE}\)\): MAE of SOC estimation; \(RDT\(_{AAE}\)\): AAE of RDT prediction; \(PT\): prognostic algorithm calculation time given by second.

In order to fully verify the performance of the proposed method, it is also tested on different types of batteries with different capacities, ages, and materials under different loading profiles. Fig. 10 shows the battery samples, which includes Battery 1 (fresh Sanyo 14500 Li-CoO2/graphite/840 mAh), Battery 2 (Sanyo 14500 Li-CoO2/graphite/840 mAh after 200 aging cycles), Battery 3 (fresh polymer lithium-ion/502030 battery/250 mAh), and Battery 4 (fresh samsung 25R 18650/2500 mAh) respectively. For Battery 1 to 4, the AAEs of the simulated and the measured terminal voltage are 15.6 mV, 18.2 mV, 16.5 mV, and 19.4 mV, respectively. The result indicates that the SFP model is generic and is able to accurately describe the behaviors of different types of batteries with different ages, capacities, and materials.

When the SFP model for different batteries are integrated with the proposed method, Table IV summarizes the performance under DST for the 4 battery samples. The AAEs of SOC estimation and RDT prediction are less than 2% and 800 seconds, respectively, for different types of battery samples with different ages, capacities, and materials. Compared with the RDT prediction ground truth (about 2.5e4), the RDT prediction errors are less than 3.2%. It is clear that the proposed method is accurate and reliable for SOC estimation and RDT prediction.

![Battery samples](image)

**Fig. 10.** Battery samples used for comparison of the proposed method.

**TABLE IV**

<table>
<thead>
<tr>
<th>Battery Samples</th>
<th>(U_{AAE}^i) (mV)</th>
<th>(SOC_{AAE})</th>
<th>(SOC_{MAE})</th>
<th>(RDT_{AAE}) (s)</th>
<th>(PT) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Battery 2</td>
<td>18.8</td>
<td>1.79%</td>
<td>4.23%</td>
<td>557</td>
<td>0.052</td>
</tr>
<tr>
<td>Battery 3</td>
<td>16.5</td>
<td>1.64%</td>
<td>4.06%</td>
<td>471</td>
<td>0.048</td>
</tr>
<tr>
<td>Battery 4</td>
<td>19.4</td>
<td>1.97%</td>
<td>5.14%</td>
<td>756</td>
<td>0.054</td>
</tr>
</tbody>
</table>

\(U_{AAE}^i\): AAE of the simulated and measured terminal voltage.

The high computation cost of EMs. To address this issue, a novel LS-based method absorbing the philosophy of only execution “as needed” is introduced with EKF. The introduced LS-EKF greatly reduces the computation, which makes EM-based methods possible to be implemented and deployed on embedded systems. However, LS-based methods with constant-Lebesgue-length often show slow convergence speed, relatively low accuracy, and cannot accommodate the dynamic loading profile well because of the constant-Lebesgue-length. To address these issues, this paper proposes a SFP model-based ALS-DEKF method, in which Lebesgue state length is online adjusted to accommodate the changing of loading profiles. That is, the algorithm are executed more frequently when the SOC measurement changes fast and less frequently when it changes slow, which optimizes the usage of computation resource. The proposed ALS-DEKF integrates the dichotomy method and EKF to enable high convergence speed and accuracy of SOC estimation and RDT prediction. Experiments and comparison studies under DST profiles are presented to verify the effectiveness of the proposed method. More experiment cases such as different temperature conditions and different dynamic loading profiles will be our future work.

**REFERENCES**


