

# JOINT ROUTING AND PLACEMENT OF VIRTUAL NETWORK FUNCTIONS

---

Jorge Crichigno<sup>1,2</sup>, D. Oliveira<sup>3</sup>, M. Pourvali<sup>3</sup>, N. Ghani<sup>3</sup>, D. Torres<sup>2</sup>

<sup>1</sup>University of South Carolina, SC, USA

<sup>2</sup>Northern New Mexico College, NM, USA

<sup>3</sup>University of South Florida, FL, USA

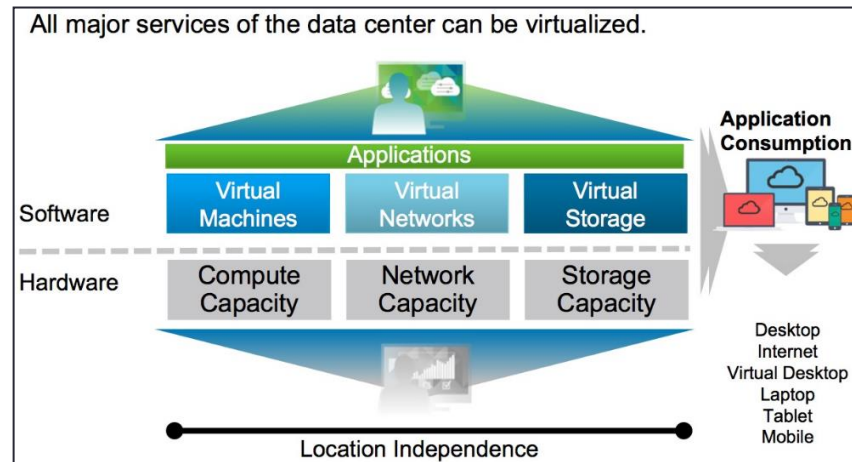
# Agenda

---

- Introduction
- Optimization model
- Numerical examples
- Concluding remarks

# Introduction

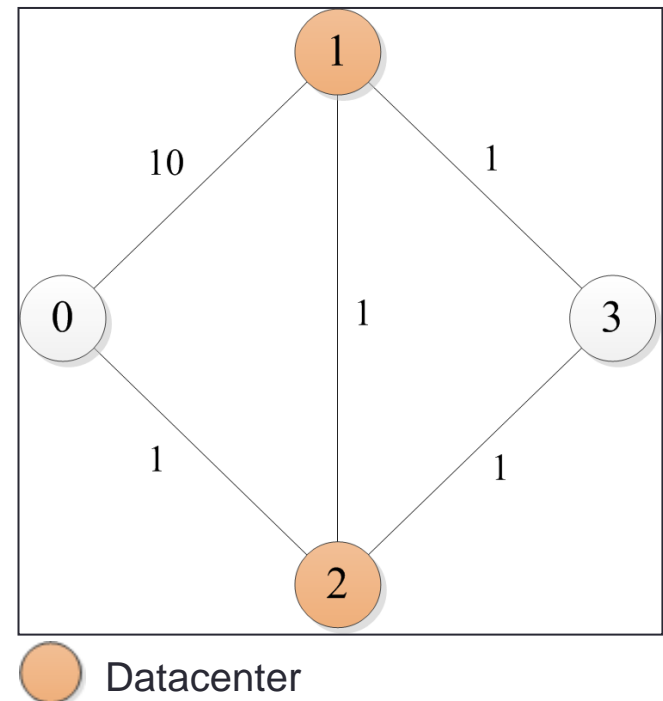
- Network Function Virtualization (NFV) is a technology that permits the implementation of Network Functions (NFs) on datacenters' commodity servers
- Network functions include
  - Firewall, access control lists
  - Routers, switches, NAT, DHCP



<http://www.vmware.com/>

# Introduction

- Consider the weighted network below
- A set of datacenter that implement particular functions
- There is a set of function  $F = \{0, 1\}$
- A client request is interested in both functions to apply them to a flow from ingress switch 0 to egress switch 3
- A datacenter  $d$  implements  $F_d \subseteq F$
- The cost and resources to implement a function are datacenter-dependent
- What should the path of the flow be, in order to minimize the routing and deployment costs?

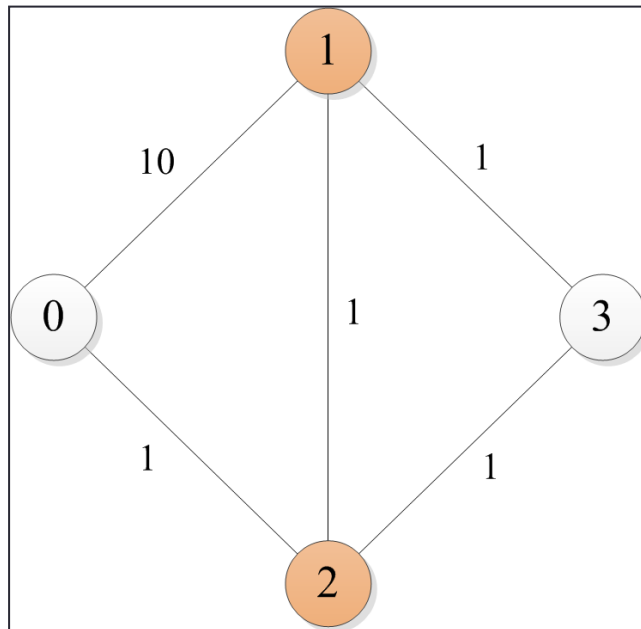


# Introduction

Example:

- Datacenter 1 implements functions 0 and 1 at costs 1 and 10
- Datacenter 2 implements functions 0 and 1 at costs 10 and 1

Function 0 at cost 1  
Function 1 at cost 10



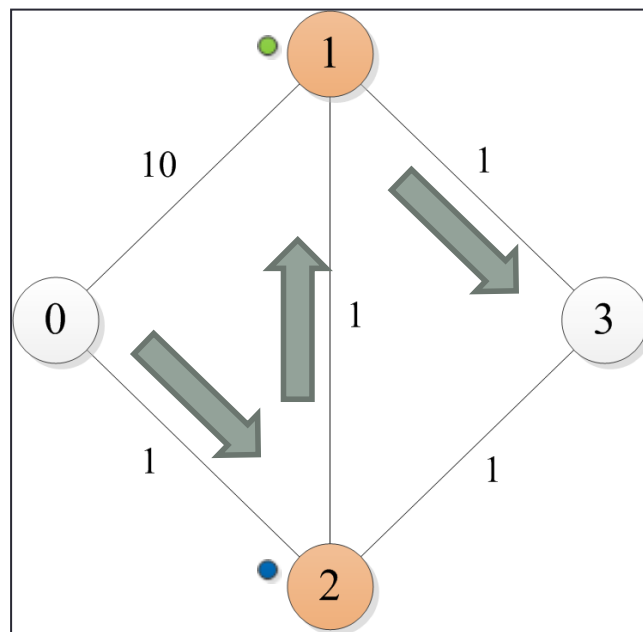
Function 0 at cost 10  
Function 1 at cost 1

# Introduction

## Example:

- Datacenter 1 implements functions 0 and 1 at costs 1 and 10
- Datacenter 2 implements functions 0 and 1 at costs 10 and 1
- The optimal solution places function 0 at datacenters 1 and 2 respectively, and route the traffic through (0, 2), (2, 1), (1,3)

Function 0 at cost 1  
Function 1 at cost 10



Function 0 at cost 10  
Function 1 at cost 1

- Function 0 instance
- Function 1 instance

# Optimization Model

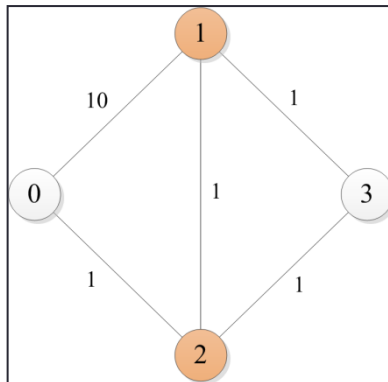
---

- The network is represented as a graph  $G = (V, E)$
- Each link  $(i, j) \in E$  has an associated cost  $c^{ij}$
- The subset  $D \subseteq V$  represents the set of datacenters
- A datacenter  $d \in D$  implements a subset of functions  $F_d \subseteq F$
- Each request  $r \in R$  is characterized by a 3-tuple  $(src_r, dst_r, F_r)$
- A datacenter has a set of resources  $W = \{w_{d,1}, w_{d,2}, \dots, w_{d,m}\}$
- To implement function  $i \in F_d$ , the datacenter uses  $w_{d,1}^i, w_{d,2}^i, \dots, w_{d,m}^i$
- The setup cost of an instance  $i \in F_d$  is  $c_d^i$
- Each instance  $i \in F_d$  can serve up to  $\lambda_d^i$  requests
- Variable  $x_{r,d}^i$  indicates whether datacenter  $d$  serves function  $i \in F_r$  requested by  $r \in R$
- Variable  $y_d^i$  indicates the number of instances of function  $i$  at  $d$
- Variable  $l_r^{ij}$  indicates whether link  $(i, j) \in E$  is used by flow  $r \in R$

# Optimization Model

- The objective is the maximization of the number of satisfied network functions (NFs)

$$\text{Max } F_1 = \sum_{r \in R} \sum_{i \in F_d} \sum_{d \in D} x_{r,d}^i = x_{0,1}^0 + x_{0,1}^1 + x_{0,2}^0 + x_{0,2}^1$$





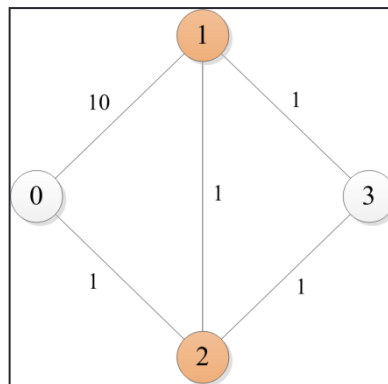
# Optimization Model

- The objective is the maximization of the number of satisfied network functions (NFs)

$$\text{Max } F_1 = \sum_{r \in R} \sum_{i \in F_d} \sum_{d \in D} x_{r,d}^i = x_{0,1}^0 + x_{0,1}^1 + x_{0,2}^0 + x_{0,2}^1$$

- Minimization of the NF deployment cost

$$\text{Max } -F_2 = \sum_{d \in D} \sum_{i \in F_d} c_d^i y_d^i = y_1^0 + 10y_1^1 + 10y_2^0 + 1y_2^1$$

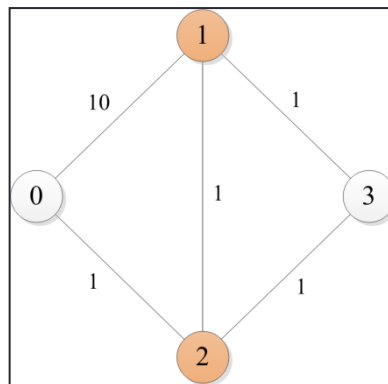


# Optimization Model

- Minimization of the routing cost

$$\text{Max - F3} = \sum_{r \in R} \sum_{(i,j) \in E} c^{ij} l_r^{(i,j)} =$$

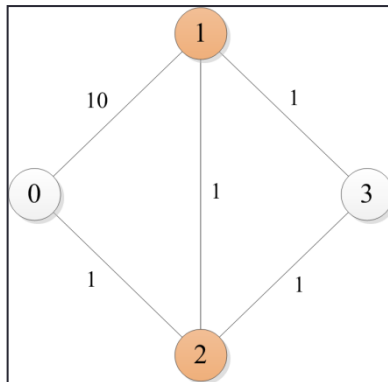
$$10l_0^{(0,1)} + l_0^{(0,2)} + 10l_0^{(1,0)} + l_0^{(1,2)} + l_0^{(1,3)} + l_0^{(2,0)} + l_0^{(2,1)} + l_0^{(2,3)} + l_0^{(3,1)} + l_0^{(3,2)}$$



# Optimization Model

- Requested functions 0 and 1 are only implemented in one datacenter

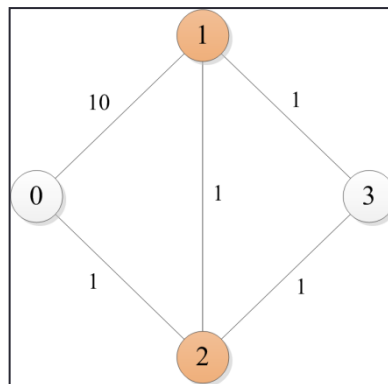
$$\sum_{d \in D} x_{r,d}^i \leq 1 \quad \longrightarrow \quad \begin{array}{ll} x_{0,1}^0 + x_{0,2}^0 \leq 1 & \text{Function 0} \\ x_{0,1}^1 + x_{0,2}^1 \leq 1 & \text{Function 1} \end{array}$$



# Optimization Model

- The total amount of resources (memory, CPU, storage) is limited at each datacenter
- E.g., 15 and 20 storage units used by an instance of function 0 and 1 respectively at datacenter 1. Datacenter has 100 storage units

$$\sum_{i \in F_d} w_{d,j}^i y_d^i \leq w_{d,j} \quad \longrightarrow \quad 15y_1^0 + 20y_1^1 \leq 100 \quad \text{Datacenter 1, storage resource}$$



# Optimization Model

- There is a path from the ingress switch 0 to egress switch 3

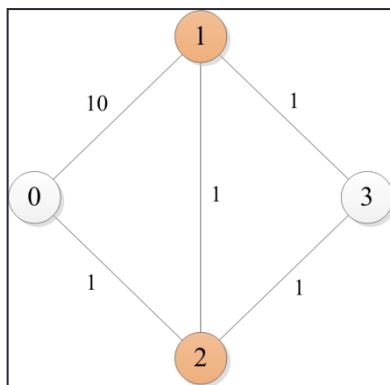
$$\text{Node 0:} \quad \left( l_0^{(0,1)} + l_0^{(0,2)} \right) - \left( l_0^{(1,0)} + l_0^{(2,0)} \right) = 1$$

$$\text{Node 1:} \quad \left( l_0^{(1,0)} + l_0^{(1,2)} + l_0^{(1,3)} \right) - \left( l_0^{(0,1)} + l_0^{(2,1)} + l_0^{(3,1)} \right) = 0$$

$$\text{Node 2:} \quad \left( l_0^{(2,0)} + l_0^{(2,1)} + l_0^{(2,3)} \right) - \left( l_0^{(0,2)} + l_0^{(1,2)} + l_0^{(3,2)} \right) = 0$$

$$\text{Node 3:} \quad \left( l_0^{(3,1)} + l_0^{(3,2)} \right) - \left( l_0^{(1,3)} + l_0^{(2,3)} \right) = -1$$

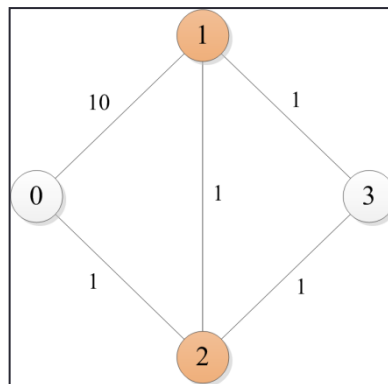
$$\sum_{j:(i,j) \in E} l_{ij}^r - \sum_{j:(j,i) \in E} l_{ji}^r = \begin{cases} -1; & i = dst_r, src_r \neq dst_r \\ 1; & i = src_r, src_r \neq dst_r \\ 0; & \text{otherwise.} \end{cases}$$



# Optimization Model

- If a function 0 is placed at datacenter 1, then the path from the ingress switch 0 to egress switch 3 must include datacenter 1

$$l_0^{(1,0)} + l_0^{(1,2)} + l_0^{(1,3)} \geq x_{0,1}^0$$



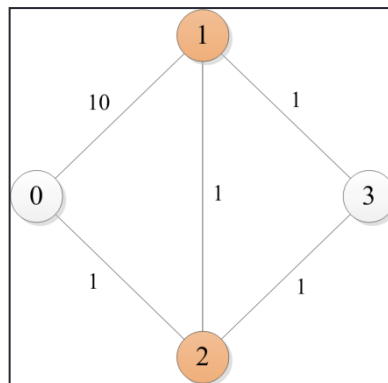
# Optimization Model

- If a function 0 is placed at datacenter 1, then the path from the ingress switch 0 to egress switch 3 must include datacenter 1

$$l_0^{(1,0)} + l_0^{(1,2)} + l_0^{(1,3)} \geq x_{0,1}^0$$

- If a function 1 is placed at datacenter 1, then the path from the ingress switch 0 to egress switch 3 must include datacenter 1

$$l_0^{(1,0)} + l_0^{(1,2)} + l_0^{(1,3)} \geq x_{0,1}^1$$



# Optimization Model

---

- If a function 0 is placed at datacenter 1, then the path from the ingress switch 0 to egress switch 3 must include datacenter 1

$$l_0^{(1,0)} + l_0^{(1,2)} + l_0^{(1,3)} \geq x_{0,1}^0$$

- If a function 1 is placed at datacenter 1, then the path from the ingress switch 0 to egress switch 3 must include datacenter 1

$$l_0^{(1,0)} + l_0^{(1,2)} + l_0^{(1,3)} \geq x_{0,1}^1$$

$$\sum_{(d,j) \in E} l_r^{dj} \geq x_{r,d}^i$$



# Optimization Model

- Variables  $x_{r,d}^i$ ,  $y_d^i$ ,  $l_{ij}^r$  are binary, integer, and real – NP hard
- For large instances of the problem, finding the optimal solution is not practical

$$\max F = w_1 \sum_{r \in R} \sum_{i \in F_r} \sum_{d \in D | i \in F_d} x_{r,d}^i - w_2 \sum_{d \in D} \sum_{i \in F_d} c_d^i y_d^i - w_3 \sum_{r \in R} \sum_{(i,j) \in E} c^{ij} l_r^{ij} \quad (1)$$

$$\sum_{d \in D} x_{r,d}^i \leq 1 \quad r \in R, i \in F_r \quad (2)$$

$$x_{r,d}^i \leq y_d^i \quad r \in R, i \in F_r, d \in D | i \in F_d \quad (3)$$

$$\sum_{i \in F_d} w_{d,j}^i y_d^i \leq w_{d,j} \quad d \in D, r \in R, j \in \{1, 2, \dots, |W_d|\} \quad (4)$$

$$\sum_{r \in R} x_{r,d}^i \leq \lambda_d^i y_d^i \quad d \in D, i \in F_d \quad (5)$$

$$\sum_{j:(i,j) \in E} l_{ij}^r - \sum_{j:(j,i) \in E} l_{ji}^r = \begin{cases} -1; & i = dst_r, src_r \neq dst_r \\ 1; & i = src_r, src_r \neq dst_r \\ 0; & \text{otherwise.} \end{cases} \quad i \in V, r \in R \quad (6)$$

$$\sum_{(d,j) \in E} l_r^{dj} \geq x_{r,d}^i \quad r \in R, i \in F_r, d \in D | i \in F_d \quad (7)$$

$$x_{r,d}^i \in \{0, 1\} \quad r \in R, i \in F_r, d \in D | i \in F_d \quad (8)$$

$$y_d^i \in Z^+ \quad d \in D, i \in F_d \quad (9)$$

$$l_r^{ij} \in \{0, 1\} \quad r \in R, (i, j) \in E \quad (10)$$

# Greedy Approach

- Greedy approach based on Dijkstra algorithm

---

## Algorithm 1 Greedy Routing and Placement of NFs

---

```

1. INPUT:  $G(V, E)$ ,  $c^{ij} \forall (i, j) \in E$ ,  $R, F, D$ 
2. OUTPUT:  $x_{r,d}^i, y_d^i, l_r^{ij}$  values
3. set  $x_{r,d}^i = 0, y_d^i = 0, l_r^{ij} = 0$  for all  $r \in R, i \in F_r, d \in D, (i, j) \in E$ 
4. for all  $r \in R$  do
5.    $D(r) = \{\}$ 
6.    $k = 1$ 
7.   for all  $i \in F_r$  do
8.      $d_k =$  datacenter that implements  $i$  at minimum cost and has enough resources
       to serve an additional request
9.     update resources of  $d_k$ 
10.    update  $y_{d_k}^i$ 
11.    set  $x_{r,d_k}^i = 1$ 
12.     $D(r) = D(r) \cup d_k$ 
13.     $k = k + 1$ 
14.  end for
15. end for
16. for all  $r \in R$  do
17.    $src = src_r$ 
18.    $C(r) = \{src\}$ 
19.   for  $k = 1$  to  $|D(r)|$  do
20.     $dst = d_k$ 
21.    if  $d_k \ni C(r)$  then
22.      $SP = Dijkstra(src, dst)$ 
23.     set  $l_r^{ij} = 1$  for all link  $(i, j) \in SP$ 
24.      $C(r) = C(r) \cup d_k$ 
25.      $C(r) = C(r) \cup j$ , for all datacenter  $j \in SP, j \in D(r)$ 
26.    end if
27.     $src = dst$ 
28.  end for
29.   $dst = dst_r$ 
30.   $SP = Dijkstra(src, dst)$ 
31.  set  $l_r^{ij} = 1$  for all  $(i, j) \in SP$ 
32. end for
33. return  $x_{r,d}^i, y_d^i, l_r^{ij}$ 

```

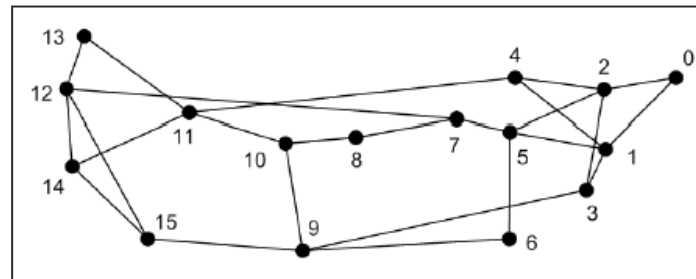
---

Placement of network functions,  
one request at a time

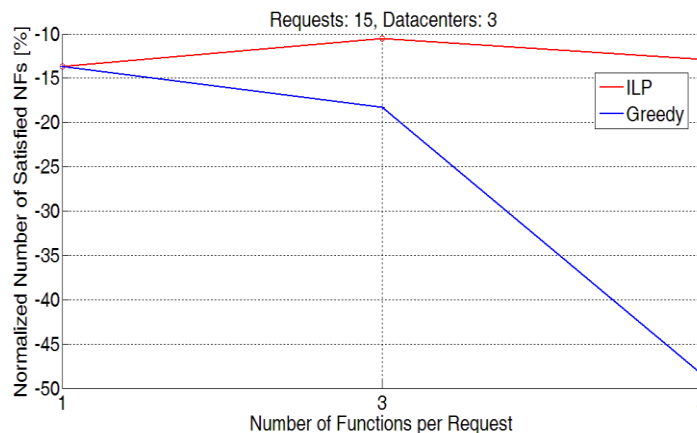
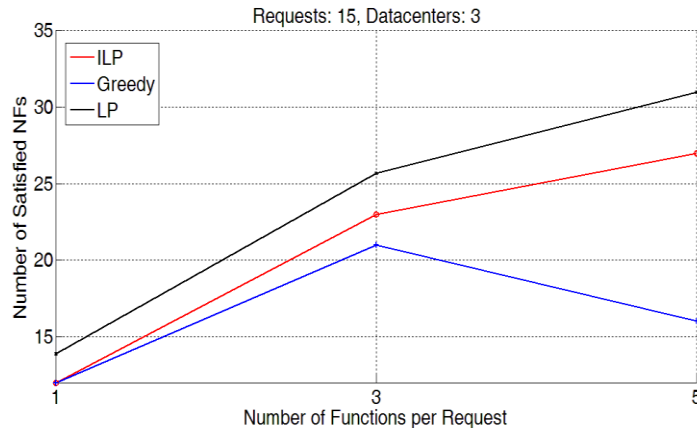
Routing of flows through datacenters  
implementing the functions, one request  
at a time

# Numerical Examples

- The number of types of resources at a datacenter was set to three (e.g., RAM, storage, CPU)
- The amount of resources of a type at a datacenter is uniform in  $[\cdot33, 300]$
- There are five network functions; each datacenter implements three functions
- The amount of resources of a type needed for an instance of a function is uniform in  $[0, 100]$
- The cost of instantiate a function is uniform in  $[0, 100]$
- Datacenters were randomly located in the topology below



# Numerical Example 1

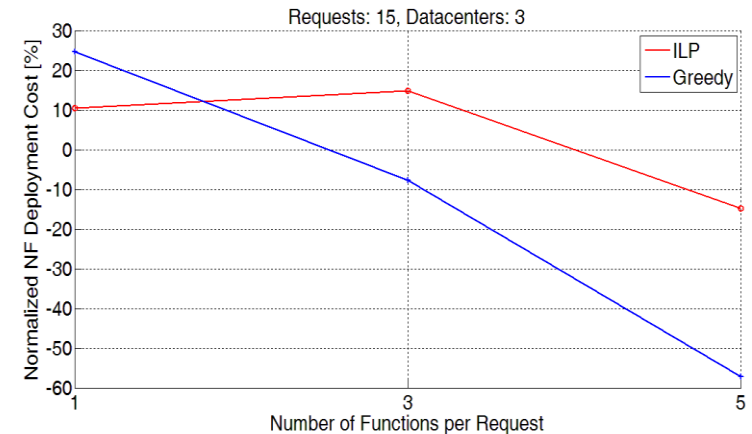
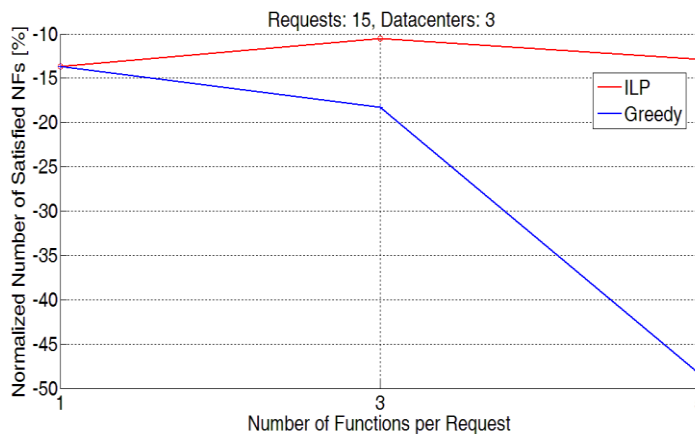
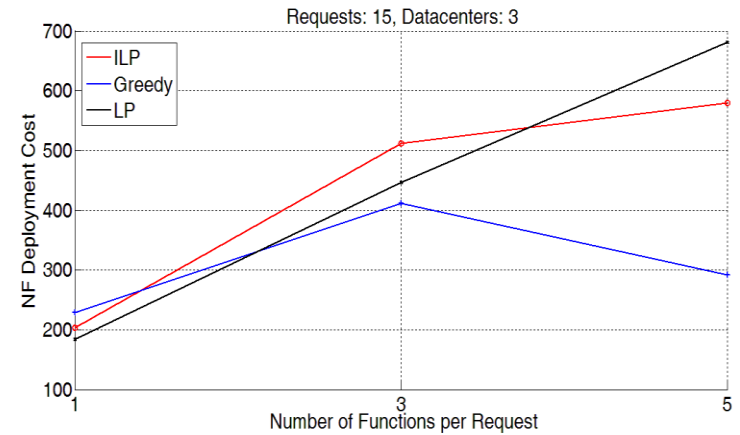
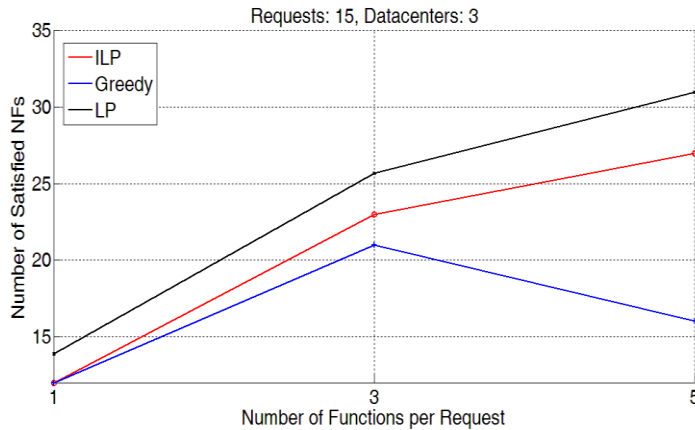


$$\text{Gap} = \frac{ov_{LP} - ov_{alg}}{ov_{LP}}$$

where  $ov_{LP}$  is the optimal value obtained with the LP scheme, and  $ov_{alg}$  is the optimal value obtained with the ILP or greedy heuristic.

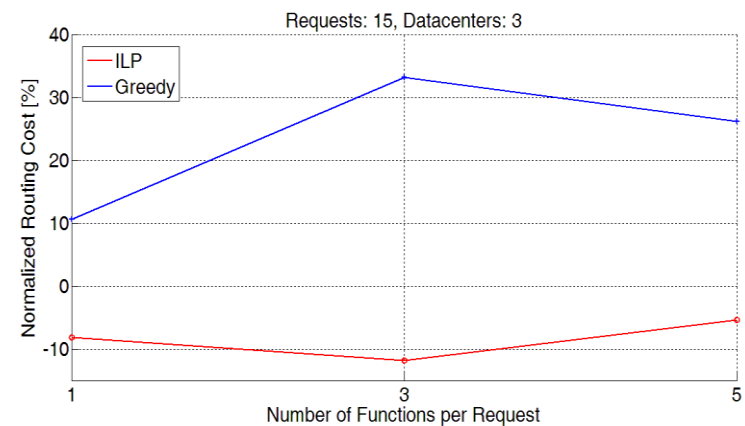
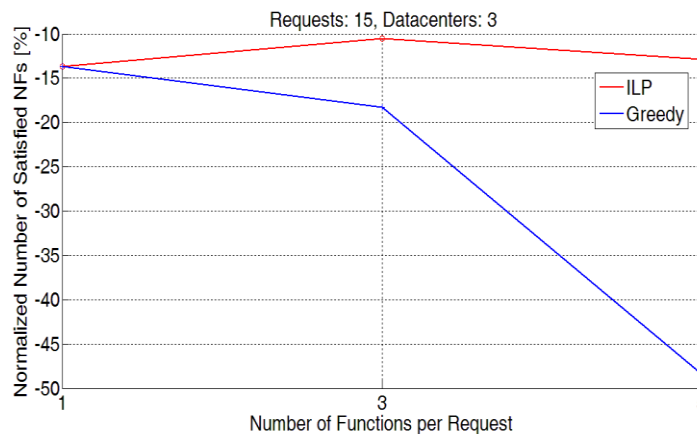
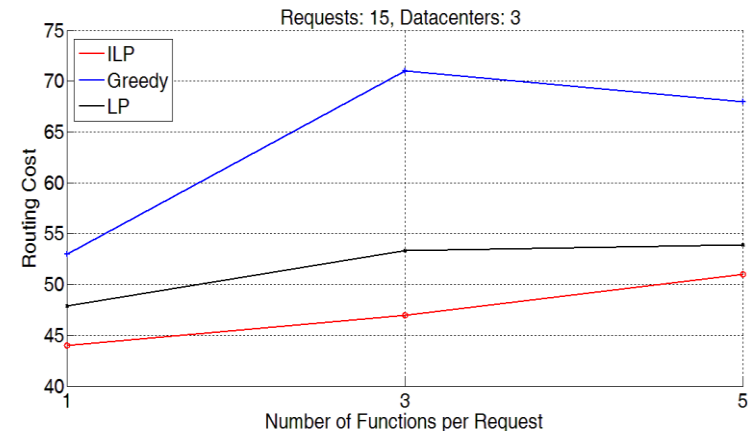
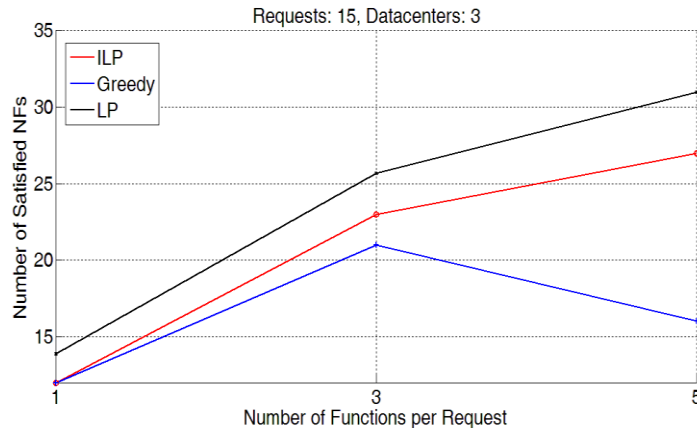
- When there is a small number of datacenters (3) and multiple requests (15), ILP has a comparable performance to that of LP; deployment cost increases with the number of function per request
- The gap of the heuristic increases with the number of function per requests; finding the optimal solution requires the evaluation of a large number of combinations

# Numerical Example 1



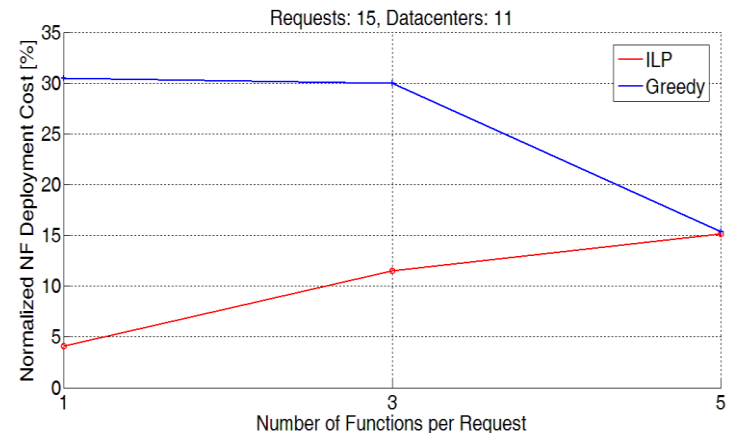
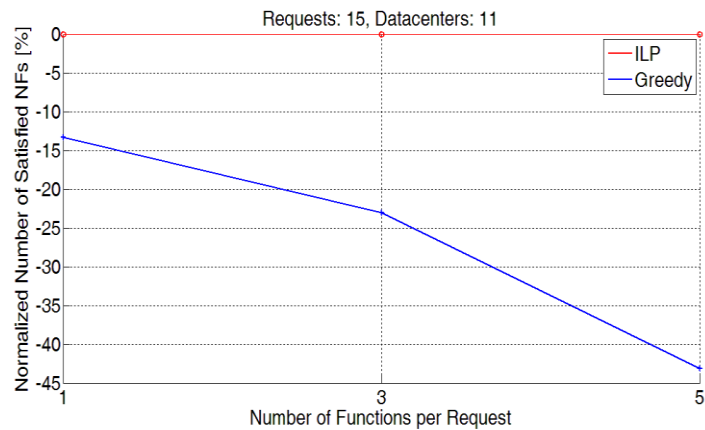
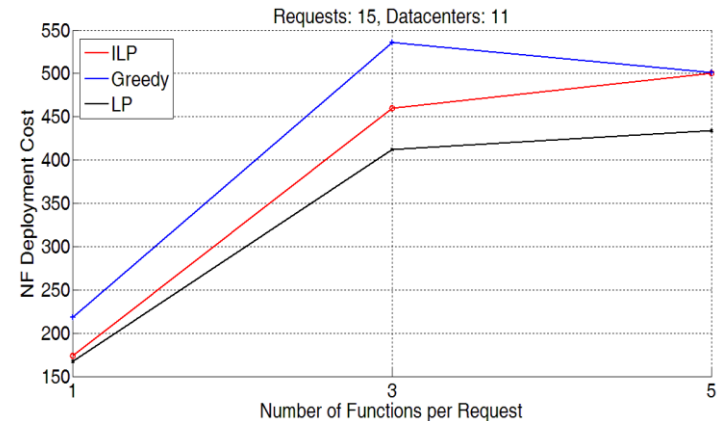
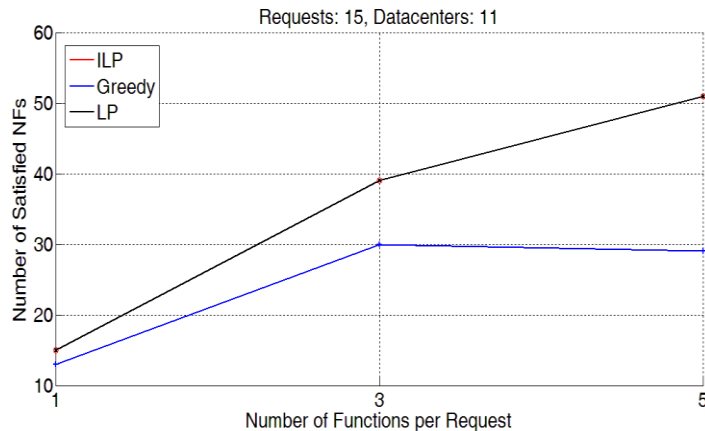
- Deployment cost increases with the number of functions per request

# Numerical Example 1



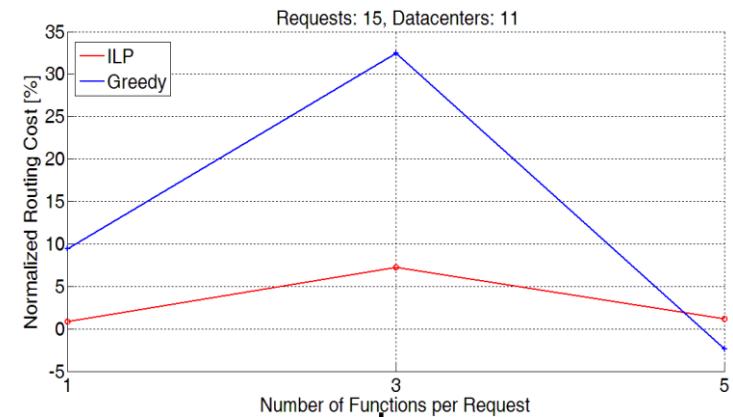
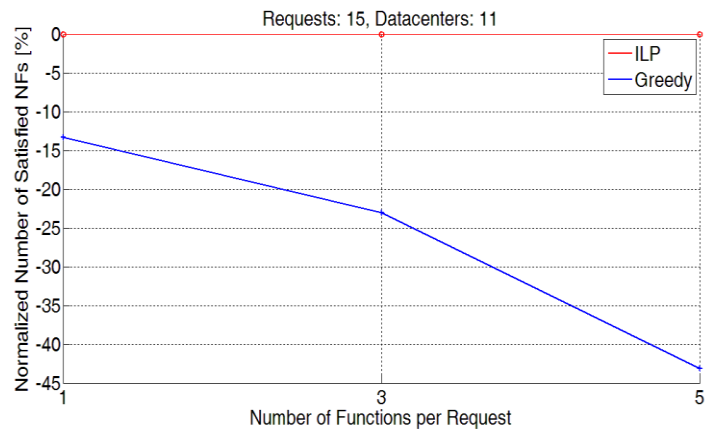
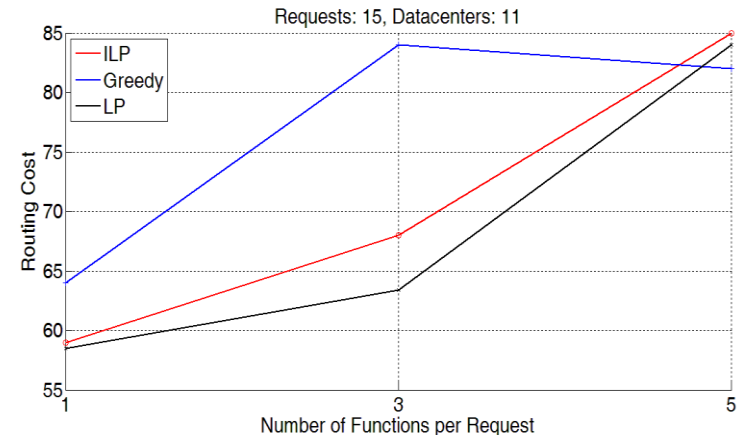
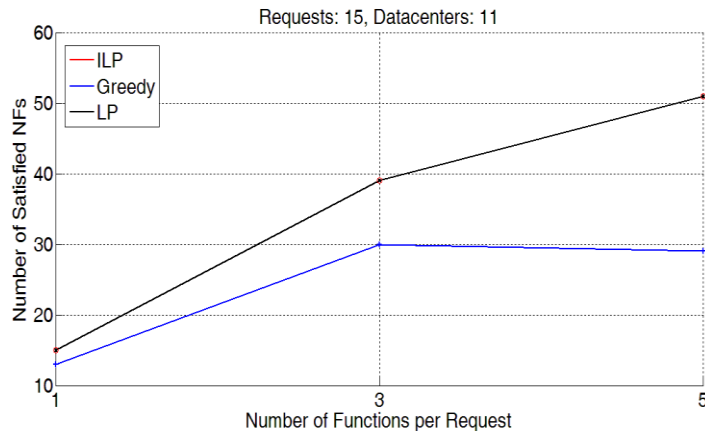
- For LP and ILP, the increase in routing cost is mostly flat; i.e., when the number of datacenters is small, routing is 'less important', because the implementation of functions are concentrated in few datacenters

# Numerical Example 2



- When there is a large number of datacenters (11) and multiple requests (15), ILP continues to have a comparable performance to LP
- Deployment cost increases substantially when the number of functions per request increases from 1 to 3. However, the increase in cost is minimal when the number of functions per request increases from 3 to 5; i.e., a single instance serves multiple requests without an increase of deployment of functions

# Numerical Example 2



- For LP and ILP, the routing cost increases with the number of function per requests; i.e., when the number of datacenters is large, routing is 'more important', because the implementation of functions are dispersed in many datacenters



# Concluding Remarks

---

- We are currently working on an optimization scheme for the joint routing and placement of virtual network functions (NFs) problem
- The proposed ILP maximizes the number of satisfied NFs while at the same time minimizes the deployment and routing costs
- A heuristics and ILP are currently being tested
- The implementation of the proposed schemes in small testbeds using ONOS SDN is being implemented

THANK YOU

---

---

# Numerical Example 3

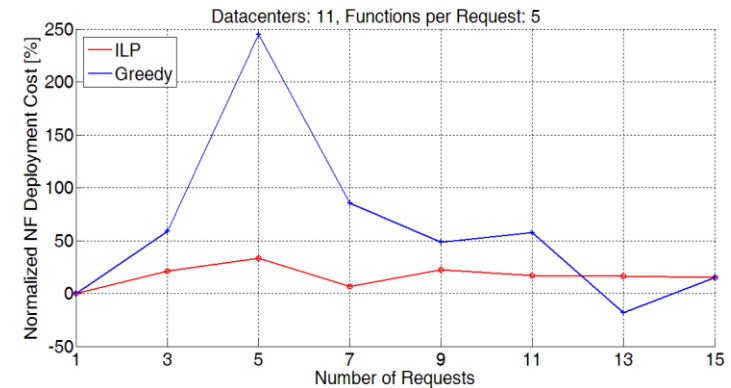
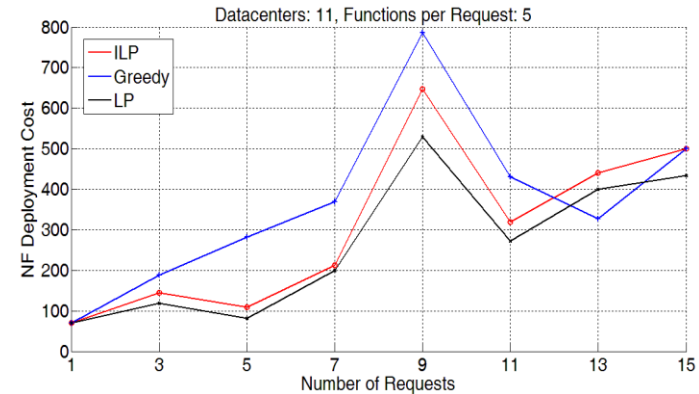
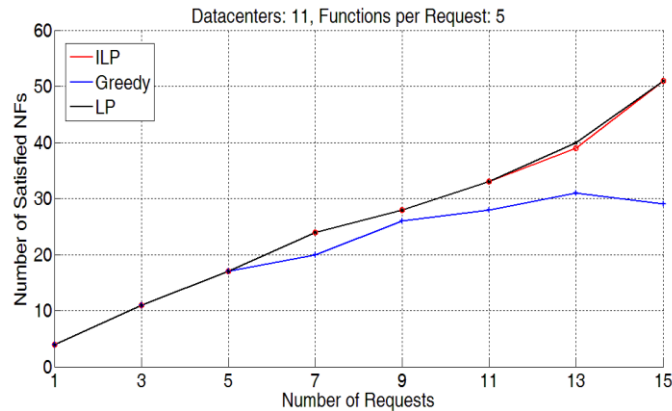


$$\text{Gap} = \frac{ov_{LP} - ov_{alg}}{ov_{LP}}$$

where  $ov_{LP}$  is the optimal value obtained with the LP scheme, and  $ov_{alg}$  is the optimal value obtained with the ILP or greedy heuristic.

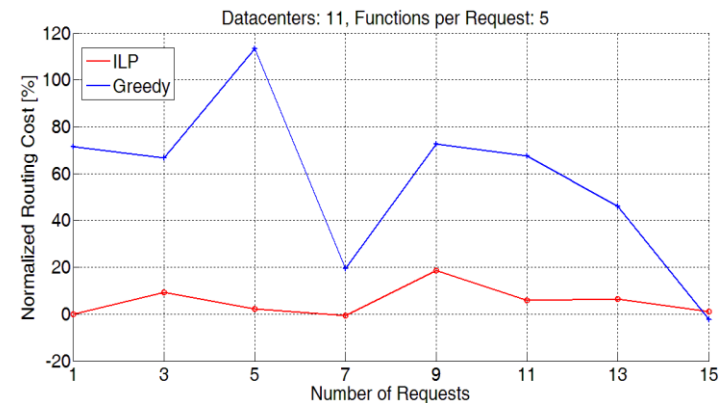
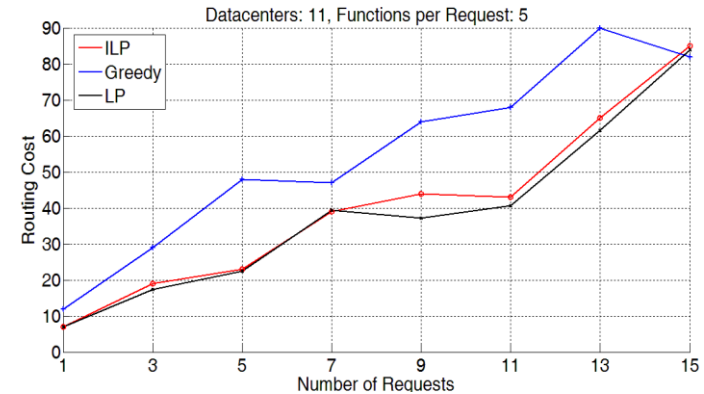
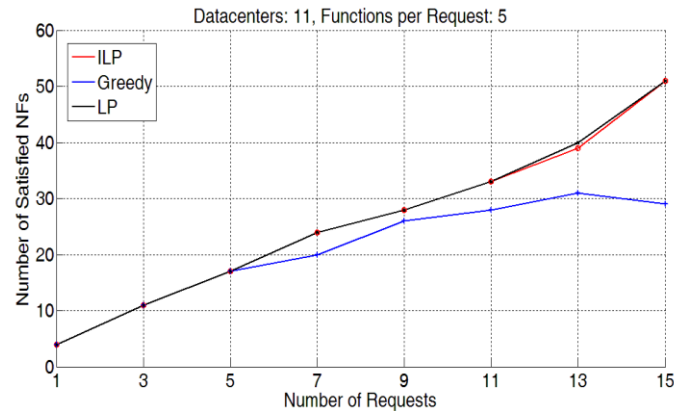
- ILP and LP performances similar; ~2% gap
- As the number of request increases, the heuristic gap substantially increases; finding the optimal solution requires the evaluation of a large number of combinations

# Numerical Example 3



- Deployment cost increases with the number of requests; ILP performance is comparable to that of LP – 'small' performance gap

# Numerical Example 3



- Routing cost increases with the number of requests; ILP performance is comparable to that of LP –'small' performance gap
- While the gap of the routing cost of the greedy approach decreases with the number of requests, the number of satisfied requests is mostly flat

# Introduction

---

## About Linear Programming

- Many of the problems for which we want algorithms are *optimization* tasks
- Optimization tasks seek a solution that (1) satisfies certain constraints and (2) is the best, with respect to a criterion
- *Linear programming* describes a broad class of optimization tasks in which both the constraints and the optimization criterion are *linear functions*

# Introduction

## About Reductions

- Sometimes a computational task is sufficiently general that any subroutine for it can also be used to solve a variety of other tasks, which at glance might seem unrelated
- Once we have an algorithm for a problem, we can use it to solve other problems

